Math 132 Spring 2008 Abrams **Exam 1** Name ________________________

Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Circle your answer when appropriate. Use the back of the sheet if necessary. As usual, notation counts! Be clear and precise with your answers, justifying responses appropriately. Make sure to include the units in your answer when warranted. You may use a standard calculator, one which does not have graphing capability or a QWERTY keyboard. Good luck!

1. (3 pt total) a. Let $f(x)$ be a continuous function defined on the closed interval $[a,b]$. Then a theorem (which we did not prove in class, but used often) says that f has both an absolute maximum and absolute minimum on $[a,b]$. The 'Closed Interval Method' tells us that these maximum and minimum values must occur at values c having one of three properties. What are these three properties?

(1)

(2)

(3)

b. Of these three properties, which one of them rarely arises in real world situations?

2. (6 pt each) For each of the functions given below, find the absolute maximum and absolute minimum values of the function on the given interval. (Make sure to clearly indicate which is which, and at what input value each happens ...)

a. $f(x) = x^4 - 2x^2 + 3$ on the interval $[-2,3]$.

b. $f(x) = \frac{x}{x^2+4}$ on the interval $[0,3]$.

c. $f(x) = x + 2\cos x$ on the interval $[0, 2\pi]$.

3. (2 pt) Complete the statement of The Mean Value Theorem. Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$ and 2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

4. (2 pt) (Fill in the blank) If $f'(x) = 0$ for all x in an interval (a, b), then

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5. (8 pt total) Throughout this problem $f(x) = xe^x$. (When appropriate, you may give your answer as intervals, or as a picture on a number line.) Make sure your answers are clear, and that your notation is precise.

a. Find the intervals on which f is increasing or decreasing.

b. Find the local maximum and minimum values for f. (Indicate which is which ...)

c. Find the intervals of concavity and the inflection points for f.

6. (10 pt total) On the given set of axes, graph the function $f(x) = 2x^3 - 3x^2 - 12x + 1$. (Here 'graph' means: (A) Domain. (B) Intercepts (always find y-intercept; find the x-intercept(s) if easy.) (C) Symmetry. (D) Asymptotes. (E) Find the intervals where f is increasing or decreasing. (F) Find the local maximum and minimum values. (G) Find the intervals of concavity, and the inflection points. (H) Then use all this information to sketch the graph. Label axes appropriately. (Note: the given 'tick marks' do not necessarily stand for one unit). Plot and label the 'important' points which arise in the analysis.) **Make sure your notation is precise!**

7. (3 pt) Use Newton's method to find the approximation x_3 of $\sqrt[4]{10}$. (Hint: consider the function $f(x) = x^4 - 10$.) Start with $x_1 = 2$. Remember that the appropriate 'iteration' formula to use in Newton's method is: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

8. (4 pt) Sketch the graph of a function that satisfies all of the given conditions.

$f'(x) > 0$ for all $x < -3$ and $x > 2$; $f'(x) < 0$ for all $-3 < x < 2$; Vertical asymptote $x = -3$;

$f'(2) = 0$; $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$

9. (1 pt each) True / False

(a) **T** **F** If c is a value at which the function f has a relative maximum or minimum, then $f'(c) = 0$.

(b) **T** **F** If c is a zero of the function $f(x)$, and we start Newton's method at any input value x_1, then Newton's method is guaranteed to produce c as a limit of the x_n values.

(c) **T** **F** If $F(x)$ and $G(x)$ are functions for which $F'(x) = G'(x)$, then $F(x) = G(x)$.

10 (3 pt) Draw a graph of a function f for which there is a value c having $f'(c) = 0$, but for which f does not have either a relative maximum or minimum at $x = c$. Label the value c on the x-axis.

11. (1 pt) If $y = f(x)$ is a continuous function on the interval $[a, b]$, then which one of these statements must ALWAYS be true?

a. If c is a value for which $f'(c) = 0$, then f must have a relative maximum or minimum at $(c, f(c))$.

b. If c is an input value for which $f(c)$ is an absolute maximum for f, then $f'(c) = 0$.

c. If c is an input value for which $f(c)$ is an absolute maximum for f, and c is not one of the endpoints a or b, then $f(c)$ is also a local maximum for f.

d. If c is an input value for which $f(c)$ is a local maximum for f, then $f(c)$ is also an absolute maximum for f.