Math 131 Fall 2009 Abrams / Holmes **Exam 1** Name ______________________

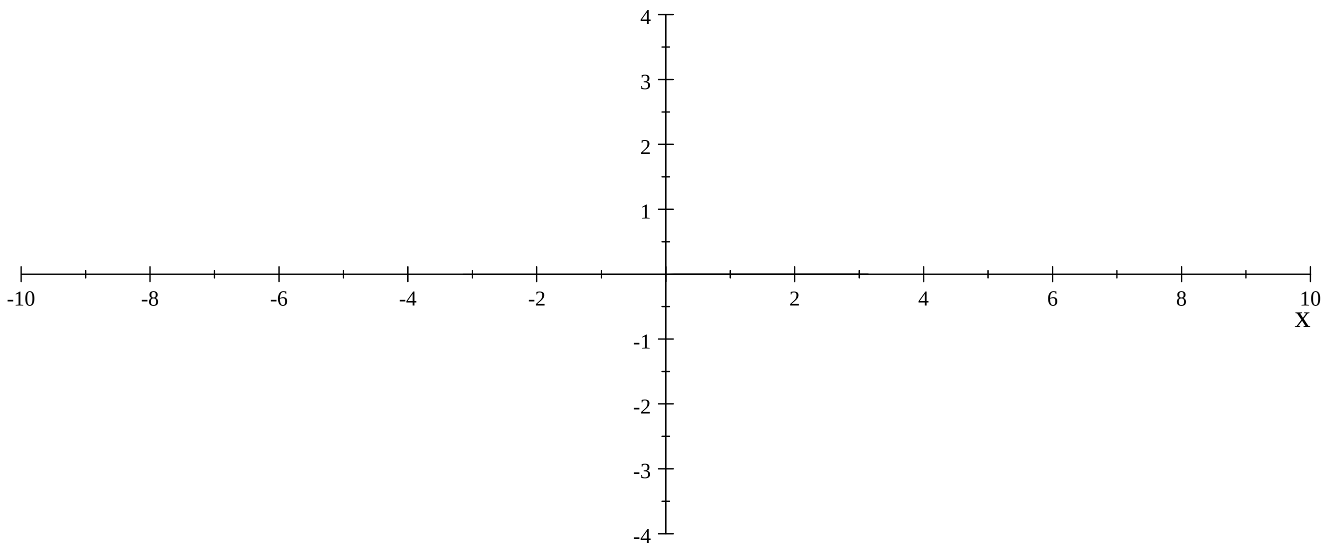
Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Only a non-graphing calculator with no QWERTY keyboard is allowed. Use the back of the sheet if you need more space. **Circle answers when appropriate**. Good luck !!

1. (2 pt) Find the equation of the line having slope 5 which passes through the point $(2,-2)$. Give your answer in $y = mx + b$ form.

2. (3 pt total) (a) Solve for x: $\sin(x) = \frac{\sqrt{2}}{2}$

(b) $\sin^{-1}(\frac{\sqrt{2}}{2}) =$

3. (3 pt) Carefully draw the function $y = 3\sin x$ on these axes. Label each of the x-intercepts with exact values involving π.



4. (1 pt each) (a) Simplify to an expression of the form e^a : $\frac{e^{-3}e^{7}}{e^{5}e^{-2}}$

(b) $\log_{16}(2) =$

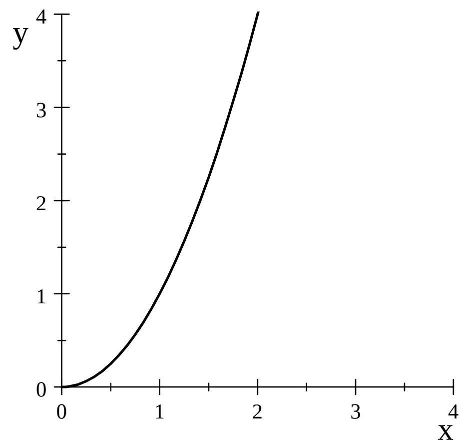
5. (3 pt) Find the inverse function for the function $f(x) = x^7 + 3$.

6. (2 pt) If $f(x) = 3x + 1$ and $g(x) = x^2 + 4x$, find and simplify $f \circ g(x)$.

7. (4 pt total) The graph of the function f is shown here. (The domain of f is $\{x \mid x \geq 0\}$.)

(a) What property of f ensures that f has an inverse function?

(b) On the same axes, sketch a graph of $f^{-1}(x)$.



8. (1 pt each) Short answer

(a) For any angle α, what is the fundamental equation which relates $\sin(\alpha)$ and $\cos(\alpha)$?

(b) Compute the exact value of $\tan(\frac{\pi}{6})$.

(c) Find the exact value of $e^{\ln(2)}$.

(d) Find the exact value of $\cos(\cos^{-1}(\frac{1}{3}))$

9. (2 pt) Give PRECISELY the meaning of the statement: $\lim_{x \to a} f(x) = L$.

10. (1 pt each) (a) (Circle the correct answer): The statement $\lim_{x \to a} f(x) = L$ intuitively means:

i. as the values of x get closer and closer to a, then eventually there is an input value whose output value equals L.

ii. All of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

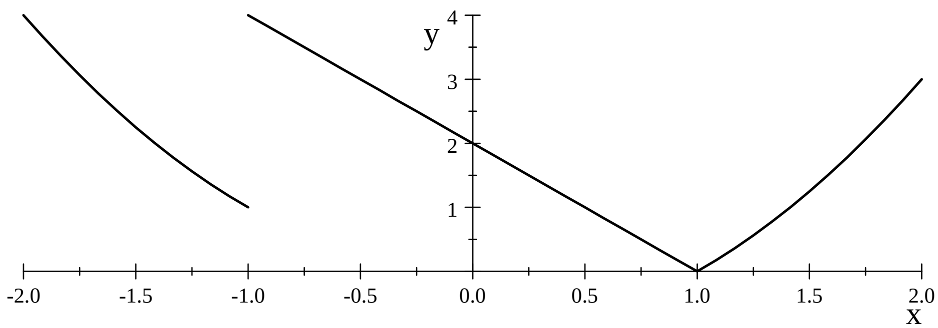
iii. the values of $f(x)$ never equal L as the values of x get closer and closer to a.

iv. Some of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

(b) **T** **F** For every function f, if $f(a) = L$ then $\lim_{x \to a} f(x) = L$.

(c) **T** **F** For every function f, if $\lim_{x \to a} f(x) = L$ then a is in the domain of f.

11. (1/2 pt each) Using the graph of the function f given here, find the indicated limits.



(a) $\lim_{x\to -1^-} f(x) =$ (b) $\lim_{x\to -1^+} f(x) =$ (c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 0^+} f(x) =$

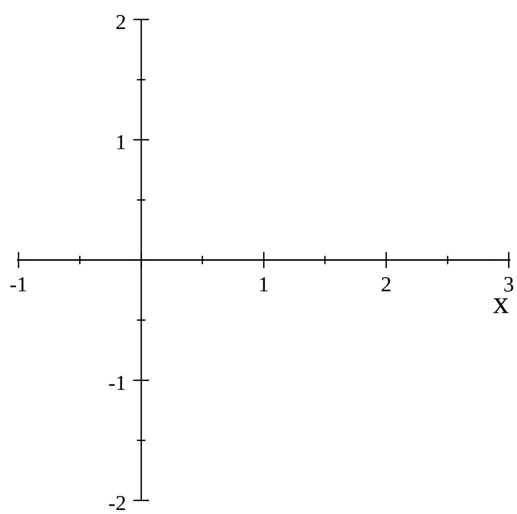
(e) $\lim_{x\to 1^+} f(x) =$ (f) $\lim_{x\to 1^-} f(x) =$ (g) $\lim_{x\to 1} f(x) =$ (h) $f(0) =$

12. (4 pt) On the left hand axes below for this problem, sketch CLEARLY a graph of a function f which has:

$\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = -1$, $\lim_{x\to 2^-} f(x) = 0$, $\lim_{x\to 2^+} f(x) = 1$, $f(2) = 0$, and $f(0) = 1$.

(Use open circles as appropriate.)

13. (2 pt) On the right hand axes below for this problem, sketch CLEARLY a graph of a function f which has: $\lim_{x\to 2} f(x) = 1$ and for which 2 is not in the domain of f. (Use open circles as appropriate.)



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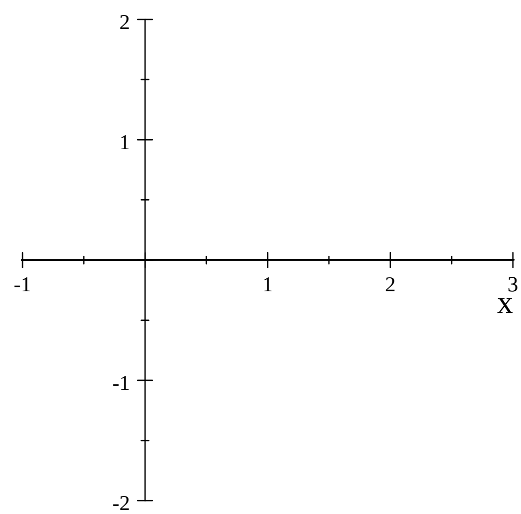
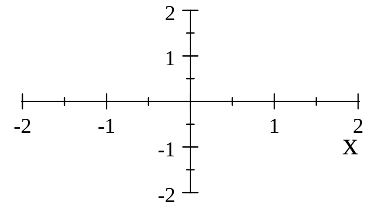


#13

14. (2 pt) Sketch clearly the graph of the function $f(x) = \frac{|x|}{x}$.



(Use open circles as appropriate.)

14. (6 pt total) Find the indicated limit. **Use the most informative of the phrases 'does not exist', ∞, or $-\infty$ whenever appropriate**. Remember to circle your answer.

(a) $\lim_{x\to -1} \dfrac{x^2 - 3x + 4}{x + 3} =$

(b) $\lim_{x\to -1} \dfrac{x^2 + 5x + 4}{x + 1} =$

.

(c) $\lim_{x\to\infty} \dfrac{x^2 + x + 4}{9 - 3x^2} =$

.

(d) $\lim_{x\to 0} \dfrac{\sin(x)}{2x} =$

.

(e) $\lim_{x\to 5} \dfrac{1}{x - 5} =$

.

15. (2 pt total). (a) Suppose $f, g,$ and k are functions which have $f(x) \le g(x) \le k(x)$ for all values of x. If $\lim_{x\to a} f(x) = L = \lim_{x\to a} k(x)$, then

$$\lim_{x\to a} g(x) = \underline{\qquad}$$

(b) What is the name of the result you used to get your answer to part (a)? (Hint: there are two possible correct answers.)

16. (1 pt) **T** **F** If f is any function with domain $[a, b]$, and $f(a) < 0$, and $f(b) > 0$, then $f(c) = 0$ for some value c in $[a, b]$.

17. (4 pt each) Compute the indicated limit. **Show all your work**, **justify each step**, **and make sure your notation is clear and precise**. Remember to circle your answer.

(a) $\lim_{h\to 0} \dfrac{(-3 + h)^2 + 7(-3 + h) + 12}{h} =$

(Problem 17, continued) (b) $\lim_{x \to 4} \dfrac{\frac{1}{x} - \frac{1}{4}}{x - 4} =$

18. (6 pt total) **Make sure to include the correct units in all parts of your answers to this question**.

(a) If a ball is thrown into the air with velocity 90 ft/sec, its height above the ground in feet after t seconds is given by $y = 90t - 16t^2$. Find the average velocity of the ball for the time period beginning when $a = 2$ and lasting

(i) 0.1 second

(ii) 0.05 second

(b) **Using limits**, find the instantaneous velocity of the ball when $a = 2$.