Advanced Macroeconomics

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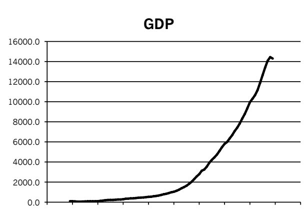
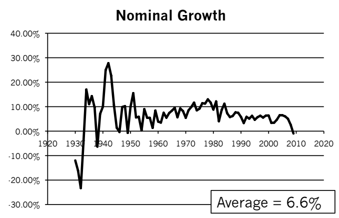
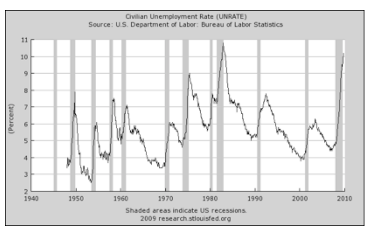
Introduction: Ch. 1

- Syllabus
- Why study macroeconomics?
  - Understanding
  - Policy recommendations
- What variables are of interest?
  - Output
    - Growth
  - Unemployment
  - Inflation

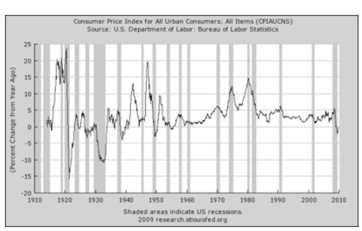
Trends and fluctuations







Average = 5.6%



Average = 3.4%

Questions

- What drives the averages?
  - Growth models
  - Policy choices
- Why does it fluctuate?

What models?

- Growth models
  - Solow
  - Endogenous growth
- Dynamic
  - IS/PC/MR
- Open economy
  - Mundell-Fleming

Exogenous growth: Ch. 13

- Solow Growth Model
  - Discrepancies in growth rates
  - Real rate of return to capital is constant
  - High investment spurs high living standards
  - Low population growth spurs high living standards

Cobb-Douglas Production

$$Y = K^{\alpha}L^{1-\alpha} \text{ where } 0 < \alpha < 1$$
$$or$$
$$\ln Y = \alpha \ln K + (1-\alpha)\ln L$$

Expressing growth

$$\frac{d\ln X}{dX} = \frac{1}{X}$$
$$\frac{d\ln X}{dt} = \frac{d\ln X}{dX} \bullet \frac{dX}{dt} = \frac{1}{X} \bullet \frac{dX}{dt}$$
$$\frac{d\ln X}{dt} = \frac{dX/dt}{X} = g_X$$
$$g_Y = \alpha g_K + (1-\alpha)g_L$$

Constant Returns to Scale

$$zY = (zK)^{\alpha}(zL)^{1-\alpha}$$
$$zY = z^{\alpha}K^{\alpha}z^{1-\alpha}L^{1-\alpha}$$
$$zY = z^{1-\alpha+\alpha}K^{\alpha}L^{1-\alpha} = zK^{\alpha}L^{1-\alpha}$$

Production in the S-model

(1) $y = \frac{Y}{L}$

(2) $Y = F(K,L)$ where F(..) displays CRS

(3) $k = \frac{K}{L}$

(4) $y = \frac{1}{L}F(K,L) = F\left(\frac{K}{L},1\right) = f(k)$

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(5) $y = \frac{K^{\alpha}}{L} \bullet \frac{L^{1-\alpha}}{L} = \frac{K^{\alpha}}{L} \bullet \frac{L}{L^{\alpha}} = \frac{K^{\alpha}}{L^{\alpha}} = \left(\frac{K}{L}\right)^{\alpha} = k^{\alpha}$

(6) $\ln y = \alpha \ln k$

(7) $g_y = \alpha g_k$

Growth in k?

(3) $k = \frac{K}{L}$

(8) $\Delta K \equiv I - \delta K$

(9) $Y = C + I$

(10) $Y = C + S$

(11) $I = S$

(12) $S = sF(K,L)$

(13) $\Delta K \equiv sF(K,L) - \delta K$

(14) $g_K = \frac{\Delta K}{K} = s\frac{Y}{K} - \delta$

$$(15)\ n = \frac{\Delta L}{L}$$

$$(16)\ \ln k \equiv \ln K - \ln L$$

$$(17)\ g_k = g_K - g_L = s\tfrac{Y}{K} - \delta - n$$

$$(18)\ g_y = \alpha\left(s\tfrac{Y}{K} - \delta - n\right)$$

$$(19)\ \tfrac{Y}{K} = APK$$

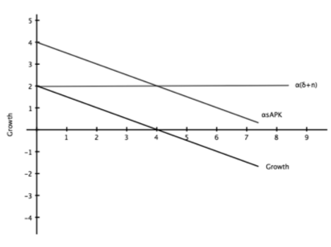
$$(20)\ g_y = \alpha\left(sAPK - \delta - n\right)$$

Derivative of APK?

$$(21)\ APK = \frac{Y}{K} = \frac{K^{\alpha}L^{1-\alpha}}{K} = K^{\alpha-1}L^{1-\alpha}$$

$$(22)\ APK' = (\alpha - 1)K^{\alpha-2}L^{1-\alpha} < 0$$

Graphing the model



Initial implications

- Growth dynamic
- Steady state

Examples

Extensions

- Population growth rate change
- Raising the savings rate
- Technological change
- “Golden Rule” savings rate

Technological change

(1') $\hat{y} = \frac{Y}{AL}$

(2') $Y = F(K, AL)$ where F(..) displays CRS

(3') $\hat{k} = \frac{K}{AL}$

(7') $g_{\hat{y}} = \alpha g_{\hat{k}}$

(16') $\ln k \equiv \ln K - \ln A - \ln L$

(17') $g_{\hat{k}} = g_K - g_A - g_L = s\frac{Y}{K} - \delta - x - n$

(20') $g_{\hat{y}} = \alpha\left(sAPK^{*} - \delta - x - n\right)$

Effect on the curves?

- APK* and APK
- Everything up!

Golden Rule

(23) max $c = \frac{C}{L}$

(24) $c = (1 - s)y = y - sy$

(25) Steady state: $sAPK = \delta + n$

(26) $c = y - \frac{1}{APK}(\delta + n)y = y - \frac{K}{Y}(\delta + n)y$

(27) $c = y - \frac{K/L}{Y/L}(\delta + n)y = y - \frac{k}{y}(\delta + n)y$

(28) $c = y - k(\delta + n) = f(k) - k(\delta + n)$

(29) $\frac{d}{dk}c(k) = f'(k) - (\delta + n)$ [k_G satisfies = 0 condition]

(25') $s_G = \frac{k_G}{f(k_G)}(\delta + n)$

Discussion

- Is all savings good?
- Implication of low savings?
- Examples

Endogenous growth: Ch. 14

- Only the first portion
- Extends the Solow model
  - Knowledge spillovers
  - Human capital
  - R&D

Knowledge spillovers

- The Silicon Valley case
  - "Nationwide economy of scale"
- The model

$$Y_i = K_i^{\alpha}(AL_i)^{1-\alpha}$$
$$A_i = A_0 K^{\eta}$$
$$A = A_0 K^{\eta}$$
$$Y = K^{\alpha}(AL)^{1-\alpha} = A_0^{1-\alpha} K^{\eta(1-\alpha)+\alpha} L^{1-\alpha}$$

- If $\eta = 1$

$$Y = A_0^{1-\alpha} K L^{1-\alpha}$$
$$\ln Y = (1-\alpha)\ln A_0 + \ln K + (1-\alpha)\ln L$$
$$g_Y = g_K + (1-\alpha)n$$

g_K and growth?

$$g_y = g_Y - n = g_K + (1-\alpha)n - n = g_K - \alpha n$$
$$g_K = s\frac{Y}{K} - \delta = s\frac{A_0^{1-\alpha} K L^{1-\alpha}}{K} - \delta$$
$$g_K = sA_0^{1-\alpha} L^{1-\alpha} - \delta$$
$$g_y = sA_0^{1-\alpha} L^{1-\alpha} - \delta - \alpha n$$

Human capital

- Lucas formulation

$$\dot{h} = ch_t(1-u_t)$$

  - c: scaling factor
  - h_t: human capital at time t
  - u_t: percentage of labor time spent working
- Per capita output

$$y_t = Ak_t^{\alpha}(u_t h_t)^{1-\alpha}$$
$$y_t = Au_t^{1-\alpha}k_t^{\alpha}h_t^{1-\alpha} = \Lambda k_t^{\alpha}h_t^{1-\alpha}$$

Growth in the Lucas model

$$\ln y_t = \ln \Lambda + \alpha \ln k_t + (1-\alpha)\ln h_t$$
$$g_y = \alpha g_k + (1-\alpha)g_h$$

- Growth in h?

$$g_h = \frac{\dot{h}}{h} = c(1-u)$$
$$g_y = \alpha g_k + (1-\alpha)c(1-u)$$

- g_k?

$$\dot{k}_t = sy_t - (\delta+n)k_t$$
$$\dot{k}_t = s\Lambda k_t^{\alpha}h_t^{1-\alpha} - (\delta+n)k_t$$
$$g_k = \frac{\dot{k}_t}{k} = s\Lambda k_t^{\alpha-1}h_t^{1-\alpha} - (\delta+n) = s\Lambda\left(\frac{h_t}{k_t}\right)^{1-\alpha} - (\delta+n)$$

g_y?

- Depends on growth of h_t and k_t
  - >; accelerating growth
  - <; decelerating growth
  - =; steady state growth

$$g_y = \alpha s\Lambda\left(\frac{h_t}{k_t}\right)^{1-\alpha} - (\delta+n) + (1-\alpha)c(1-u)$$

The R&D model

- Productive labor and research labor

$$Y_t = K_t^{\alpha} \left(A_t L_t^{Y}\right)^{1-\alpha}$$
$$\Delta A_t = cA_t^{\eta} L_t^{R}$$

Assume $\eta = 1$

$$g_A = cL_t^{R}$$

If $0 < \eta < 1$

$$g_A = cA_t^{\eta-1} L_t^{R}$$

Growth?

$$\ln Y_t = \alpha \ln K_t + (1-\alpha)\left(\ln A_t + \ln L_t^{Y}\right)$$
$$g_Y = \alpha g_K + (1-\alpha) g_A + (1-\alpha) n^{Y}$$

Assume $n^{Y} = n$

$$g_y = \alpha (g_K - n) + (1-\alpha) g_A$$
$$g_y = \alpha g_k + (1-\alpha) g_A$$

If $g_k = 0$,

$$g_y = (1-\alpha) g_A$$

Is $g_A > 0$?

Yes, for all A_t and $L_t^{R} > 0$.

IS/LM: Ch. 2

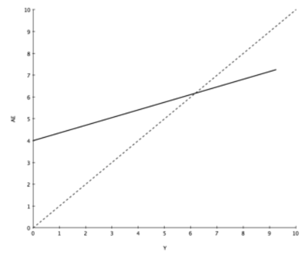
- IS: goods market equilibrium

$$Y = C + I + G$$
$$C = c_A(W) + c_Y(Y - T_0 - tY)$$
$$I = I(r; A(\text{expectations})) = A - ar$$

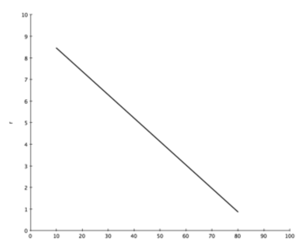
$$Y = c_A(W) + c_Y(Y - T_0 - tY) + A - ar + G$$

$$Y = \frac{1}{1 - c_Y(1-t)}\left(c_A(W) - c_Y T_0 + A - ar + G\right)$$

45° diagram



IS curve



Multiplier

$$\text{multiplier} = \mu = \frac{1}{1 - c_Y (1 - t)}$$

$$\text{exogenous constants} = \kappa = c_A (W) - c_Y T_0 + A + G$$

$$Y = \mu (\kappa - ar) = \mu\kappa - a\mu r$$

$$a\mu r = \mu\kappa - Y$$

$$r = \frac{1}{a\mu} \mu\kappa - \frac{1}{a\mu} Y = \frac{1}{a} \kappa - \frac{1}{a\mu} Y$$

LM: money market equilibrium

$$\frac{M_s}{P} = L\,(Y,\,i) = \mathcal{L} - \iota i + \frac{1}{v}\,Y$$

Which interest rate?

$$1 + r = (1 + i)\,\frac{P}{P_e}$$

$$\pi_e = \frac{P_e - P}{P}$$

$$\frac{P}{P_e} = \frac{1}{1 + \pi_e}$$

$$1 + r = \frac{1}{1 + \pi_e}\,(1 + i)$$

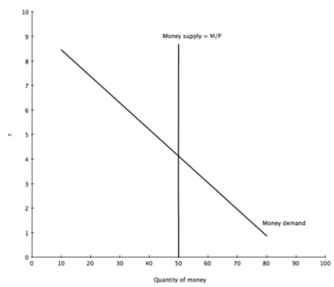
$$\text{As } \pi_e \Longrightarrow 0,\ \ r \approx i$$

LM (again)

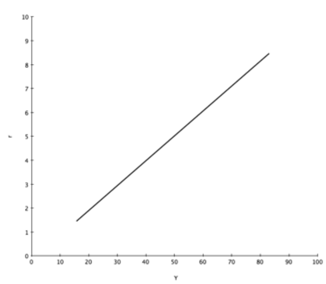
$$\frac{M_s}{P} = \mathcal{L} - \iota r + \frac{1}{v}\,Y$$

$$r = \frac{1}{\iota}\left(\mathcal{L} - \frac{M_s}{P}\right) + \frac{1}{\iota v}\,Y$$

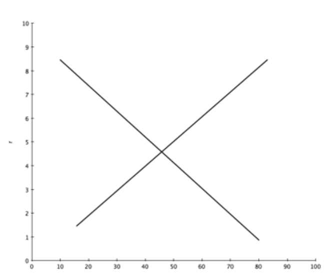
Money market



LM curve



Equilibrium



Linear algebra solution

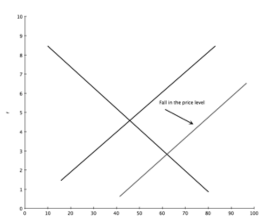
$$\mathbf{a}\mu\mathbf{r} = \mu\kappa - \mathbf{Y}$$
$$\mathbf{Y} + \mathbf{a}\mu\mathbf{r} = \mu\kappa$$
$$\frac{1}{\mu}\mathbf{Y} + \mathbf{a}\mathbf{r} = \kappa$$

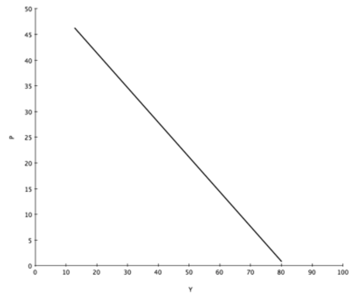
$$\frac{1}{\mathbf{v}}\mathbf{Y} - \iota\mathbf{r} = \frac{\mathbf{M_S}}{\mathbf{P}} - \mathcal{L}$$
$$\begin{pmatrix} \frac{1}{\mu} & \mathbf{a} \\ \frac{1}{\mathbf{v}} & -\iota \end{pmatrix} \begin{pmatrix} \mathbf{Y} \\ \mathbf{r} \end{pmatrix} = \begin{pmatrix} \mu\kappa \\ \frac{\mathbf{M_S}}{\mathbf{P}} - \mathcal{L} \end{pmatrix}$$
$$\mathbf{Y} = \frac{|\mathbf{A_1}|}{|\mathbf{A}|}$$
$$\mathbf{r} = \frac{|\mathbf{A_1}|}{|\mathbf{A}|}$$

Examples

AS/AD: Ch. 2

- Finding the AD curve

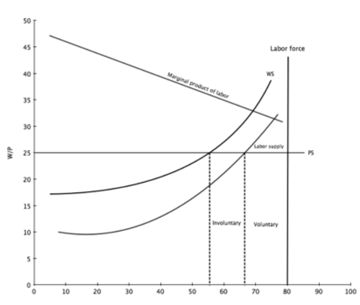




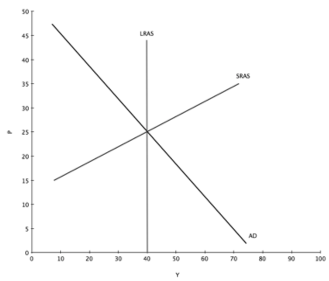
Supply side: the labor market

- Options (supply/demand)
  - Perfect competition/perfect competition
  - Imperfect competition/perfect competition
  - Perfect competition/imperfect competition

WS/PS curves



AD/AS



Examples

- Fiscal policy
- Negative supply shock