

Practice Exam - December 11, 2009

On the front page, please PRINT your NAME and your student ID number.

This is NOT a multiple choice test, so **SHOW ALL YOUR WORK**. Correct answer with no work shown will receive **zero points**, while incorrect answers with correct work shown will receive partial credit! Calculators are neither required nor allowed in this test.

1. (100 points) Given the function:

$$f(x) = 2x - 1 + \ln|e^x - 2|$$

(a) (5 points) Find the domain of $f(x)$.

(b) (2 points) Find the y -intercept, if any.

(c) (3 points) Does the function have any symmetry? Explain.

(d) (20 points) Check for vertical and horizontal asymptotes.

(d1) (10 points) Check for slant asymptotes.

(e) (20 points) Find the first derivative of $f(x)$ and increase and decrease intervals.

(f) (5 points) Find critical numbers, local max and min values.

(g) (20 points) Find the second derivative of $f(x)$ and study the concavity.

(h) (5 points) Find inflection point?

(i) (5 points) Sketch the graph of the function.

(j) (5 points) Can you use the information above to find an estimate for the x -intercepts, i.e., for the solutions of the equation $f(x) = 0$?

2. (20 points) Calculate the following integrals.

(a)

$$\int (\tan x) \ln(\cos x) dx$$

(b)

$$\int_0^4 \frac{x}{\sqrt{1+2x}} dx$$

3. (10 points) Find all discontinuities of the following function and specify what type they are.

$$f(x) = \begin{cases} \frac{x^2 - 2x - 8}{x + 2} & x < -2 \\ -\frac{18}{x - 1} & x \geq -2 \end{cases}$$

4. (10 points) Calculate the domain of

$$f(x) = \cos^{-1} \frac{1+x^2}{1+2x^2}.$$

5. (10 points) Calculate the limit

$$\lim_{x \rightarrow +\infty} \tan^{-1}(x - \sqrt{1+x^2}).$$

6. (10 points) The limit

$$\lim_{x \rightarrow 0^+} (\sin x)^x$$

is in an undetermined form (explain). Use l'Hôpital's rule to calculate it. [Hint: use the identity $f(x)^{g(x)} = e^{g(x)\ln f(x)}$ to represent the function.]

7. (10 points) Use the squeeze theorem to show that

$$\lim_{x \rightarrow 0} x^2 \cos \frac{2}{x} = 0.$$

8. **(10 points)** Use implicit differentiation to calculate the tangent line to the curve

$$\sqrt{x+y} = 1 + x^2y^2$$

at the point $(0, 1)$.

9. **(10 points)** Use the definition of the derivative as a limit to find $f'(x)$ if $f(x) = 1/(2x^2 + 1)$.

10. **(10 points)** Use the definition of limit to prove that

$$\lim_{x \rightarrow +\infty} \frac{x}{x+1} = 1.$$

11. **(10 points)** Find two numbers whose difference is 100 and whose product is minimum.