ec. 4.3

2. (a) $f(x) = x^4 - 4x - 1 \quad \Rightarrow \quad f'(x) = 4x^3 - 4 = 4(x^3 - 1) = 4(x-1)(x^2+x+1)$. So $f'(x) > 0 \quad \Leftrightarrow$ $x - 1 > 0 \quad [4(x^2+x+1) > 0] \quad \Leftrightarrow \quad x > 1$. Thus, f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.

(b) f changes from decreasing to increasing at its only critical number, $x = 1$. Thus, $f(1) = -4$ is a local minimum value.

(c) $f'(x) = 4x^3 - 4 \quad \Rightarrow \quad f''(x) = 12x^2$. $f''(x) > 0$ for all x except $x = 0$. Thus, f is CU on $(-\infty, 0)$ and $(0, \infty)$. Moreover, since f' is increasing on $(-\infty, \infty)$, f is CU on $(-\infty, \infty)$. There are no IPs.

4. (a) $f(x) = \dfrac{x^2}{x^2+3} \quad \Rightarrow \quad f'(x) = \dfrac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \dfrac{6x}{(x^2+3)^2}$. The denominator is positive so the sign of $f'(x)$ is determined by the sign of x. Thus, $f'(x) > 0 \quad \Leftrightarrow \quad x > 0$ and $f'(x) < 0 \quad \Leftrightarrow \quad x < 0$. So f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) f changes from decreasing to increasing at $x = 0$. Thus, $f(0) = 0$ is a local minimum value.

(c) $f''(x) = \dfrac{(x^2+3)^2(6) - 6x \cdot 2(x^2+3)(2x)}{[(x^2+3)^2]^2} = \dfrac{6(x^2+3)\left[x^2+3-4x^2\right]}{(x^2+3)^4} = \dfrac{6(3-3x^2)}{(x^2+3)^3} = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$.

$f''(x) > 0 \quad \Leftrightarrow \quad -1 < x < 1$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < -1$ or $x > 1$. Thus, f is CU on $(-1, 1)$ and CD on $(-\infty, -1)$ and $(1, \infty)$. There are IPs at $\left(\pm 1, \frac{1}{4}\right)$.

32. (a) $f(x) = \ln(x^4 + 27) \quad \Rightarrow \quad f'(x) = \dfrac{4x^3}{x^4+27}$. $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$, so f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

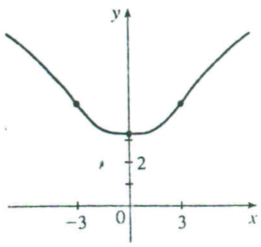
(b) $f(0) = \ln 27 \approx 3.3$ is a local minimum value.

(c) $f''(x) = \dfrac{(x^4+27)(12x^2) - 4x^3(4x^3)}{(x^4+27)^2} = \dfrac{4x^2\left[3(x^4+27) - 4x^4\right]}{(x^4+27)^2}$

$= \dfrac{4x^2(81 - x^4)}{(x^4+27)^2} = \dfrac{-4x^2(x^2+9)(x+3)(x-3)}{(x^4+27)^2}$

$f''(x) > 0$ if $-3 < x < 0$ and $0 < x < 3$, and $f''(x) < 0$ if $x < -3$ or $x > 3$. Thus, f is CU on $(-3, 0)$ and $(0, 3)$ [hence on $(-3, 3)$] and f is CD on $(-\infty, -3)$ and $(3, \infty)$. There are IPs at $(\pm 3, \ln 108) \approx (\pm 3, 4.68)$.

(d)



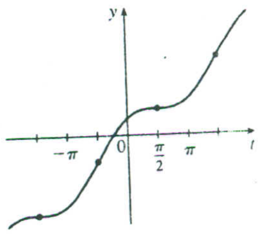
34. (a) $f(t) = t + \cos t$, $-2\pi \le t \le 2\pi \quad \Rightarrow \quad f'(t) = 1 - \sin t \ge 0$ for all t and $f'(t) = 0$ when $\sin t = 1 \quad \Leftrightarrow \quad t = -\frac{3\pi}{2}$ or $\frac{\pi}{2}$, so f is increasing on $(-2\pi, 2\pi)$.

(b) No maximum or minimum

(c) $f''(t) = -\cos t > 0 \quad \Leftrightarrow \quad t \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, so f is CU on these intervals and CD on $\left(-2\pi, -\frac{3\pi}{2}\right)$, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and $\left(\frac{3\pi}{2}, 2\pi\right)$.

There are IPs at $\left(\pm\frac{3\pi}{2}, \pm\frac{3\pi}{2}\right)$ and $\left(\pm\frac{\pi}{2}, \pm\frac{\pi}{2}\right)$.

(d)



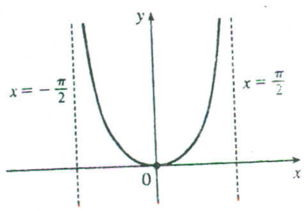
38. (a) $\lim\limits_{x \to \pi/2^-} x \tan x = \infty$ and $\lim\limits_{x \to -\pi/2^+} x \tan x = \infty$, so $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$ are VAs.

(b) $f(x) = x \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. $f'(x) = x \sec^2 x + \tan x > 0 \quad \Leftrightarrow$ $0 < x < \frac{\pi}{2}$, so f increases on $\left(0, \frac{\pi}{2}\right)$ and decreases on $\left(-\frac{\pi}{2}, 0\right)$.

(c) $f(0) = 0$ is a local minimum value.

(d) $f''(x) = 2\sec^2 x + 2x \tan x \sec^2 x > 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so f is CU on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. No IP

(e)



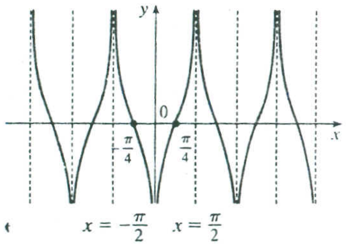
42. (a) f is periodic with period π, so we consider only $-\frac{\pi}{2} < x < \frac{\pi}{2}$. $\lim_{x\to 0} \ln(\tan^2 x) = -\infty$, $\lim_{x\to(\pi/2)^-} \ln(\tan^2 x) = \infty$, and $\lim_{x\to(-\pi/2)^+} \ln(\tan^2 x) = \infty$, so $x = 0$, $x = \pm\frac{\pi}{2}$ are VAs.

(b) $f(x) = \ln(\tan^2 x) \quad\Rightarrow\quad f'(x) = \dfrac{2\tan x \sec^2 x}{\tan^2 x} = 2\,\dfrac{\sec^2 x}{\tan x} > 0 \quad\Leftrightarrow\quad \tan x > 0 \quad\Leftrightarrow\quad 0 < x < \frac{\pi}{2}$, so f is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(-\frac{\pi}{2}, 0\right)$.

(c) No maximum or minimum

(d) $f'(x) = \dfrac{2}{\sin x \cos x} = \dfrac{4}{\sin 2x} \quad\Rightarrow\quad f''(x) = \dfrac{-8\cos 2x}{\sin^2 2x} < 0 \quad\Leftrightarrow$ $\cos 2x > 0 \quad\Leftrightarrow\quad -\frac{\pi}{4} < x < \frac{\pi}{4}$, so f is CD on $\left(-\frac{\pi}{4}, 0\right)$ and $\left(0, \frac{\pi}{4}\right)$, and CU on $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. There are IPs at $\left(\pm\frac{\pi}{4}, 0\right)$.

(e)



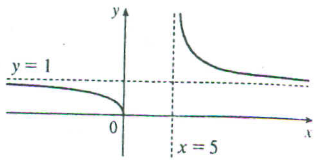
Sec. 4.4

20. $y = f(x) = \sqrt{x/(x-5)}$ **A.** $D = \{x \mid x/(x-5) \geq 0\} = (-\infty, 0] \cup (5, \infty)$. **B.** Intercepts are 0.

C. No symmetry **D.** $\lim\limits_{x \to \pm\infty} \sqrt{\dfrac{x}{x-5}} = \lim\limits_{x \to \pm\infty} \sqrt{\dfrac{1}{1-5/x}} = 1$, so $y = 1$ is a HA. $\lim\limits_{x \to 5^+} \sqrt{\dfrac{x}{x-5}} = \infty$, so $x = 5$ is a VA. **E.** $f'(x) = \dfrac{1}{2}\left(\dfrac{x}{x-5}\right)^{-1/2} \dfrac{(-5)}{(x-5)^2} = -\frac{5}{2}\left[x(x-5)^3\right]^{-1/2} < 0$, so f is decreasing on $(-\infty, 0)$ and $(5, \infty)$.

F. No extreme values **G.** $f''(x) = \frac{5}{4}[x(x-5)^3]^{-3/2}(x-5)^2(4x-5) > 0$ for $x > 5$, and $f''(x) < 0$ for $x < 0$, so f is CU on $(5, \infty)$ and CD on $(-\infty, 0)$. No IP

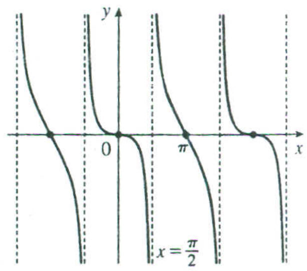
H.



28. $y = f(x) = \sin x - \tan x$ **A.** $D = \left\{x \mid x \neq (2n+1)\frac{\pi}{2}\right\}$ **B.** $y = 0 \Leftrightarrow \sin x = \tan x = \dfrac{\sin x}{\cos x} \Leftrightarrow \sin x = 0$ or $\cos x = 1 \Leftrightarrow x = n\pi$ (x-intercepts), y-intercept $= f(0) = 0$ **C.** $f(-x) = -f(x)$, so the curve is symmetric about $(0, 0)$. Also periodic with period 2π **D.** $\lim\limits_{x \to (\pi/2)^-} (\sin x - \tan x) = -\infty$ and $\lim\limits_{x \to (\pi/2)^+} (\sin x - \tan x) = \infty$, so $x = n\pi + \frac{\pi}{2}$ are VAs. **E.** $f'(x) = \cos x - \sec^2 x \leq 0$, so f decreases on each interval in its domain, that is, on $\left((2n-1)\frac{\pi}{2}, (2n+1)\frac{\pi}{2}\right)$. **F.** No extreme values

G. $f''(x) = -\sin x - 2\sec^2 x \tan x = -\sin x\,(1 + 2\sec^3 x)$. Note that $1 + 2\sec^3 x \neq 0$ since $\sec^3 x \neq -\frac{1}{2}$. $f''(x) > 0$ for $-\frac{\pi}{2} < x < 0$ and $\frac{3\pi}{2} < x < 2\pi$, so f is CU on $\left((n - \frac{1}{2})\pi, n\pi\right)$ and CD on $\left(n\pi, (n + \frac{1}{2})\pi\right)$. f has IPs at $(n\pi, 0)$. Note also that $f'(0) = 0$, but $f'(\pi) = -2$.

H.



44. $y = f(x) = \tan^{-1}\left(\dfrac{x-1}{x+1}\right)$ **A.** $D = \{x \mid x \neq -1\}$ **B.** x-intercept $= 1$, y-intercept $= f(0) = \tan^{-1}(-1) = -\frac{\pi}{4}$

C. No symmetry **D.** $\lim\limits_{x \to \pm\infty} \tan^{-1}\left(\dfrac{x-1}{x+1}\right) = \lim\limits_{x \to \pm\infty} \tan^{-1}\left(\dfrac{1-1/x}{1+1/x}\right) = \tan^{-1} 1 = \frac{\pi}{4}$, so $y = \frac{\pi}{4}$ is a HA.

Also, $\lim\limits_{x \to -1^+} \tan^{-1}\left(\dfrac{x-1}{x+1}\right) = -\dfrac{\pi}{2}$ and $\lim\limits_{x \to -1^-} \tan^{-1}\left(\dfrac{x-1}{x+1}\right) = \dfrac{\pi}{2}$.

E. $f'(x) = \dfrac{1}{1 + [(x-1)/(x+1)]^2} \dfrac{(x+1)-(x-1)}{(x+1)^2}$

$= \dfrac{2}{(x+1)^2 + (x-1)^2} = \dfrac{1}{x^2+1} > 0$

so f is increasing on $(-\infty, -1)$ and $(-1, \infty)$. **F.** No extreme values

G. $f''(x) = -2x/(x^2+1)^2 > 0 \Leftrightarrow x < 0$, so f is CU on $(-\infty, -1)$ and $(-1, 0)$, and CD on $(0, \infty)$. IP at $(0, -\frac{\pi}{4})$

H.